

1

Raccolta esercizi ***Equazioni esponenziali***

Franco Fusier - 05/2012

Equazioni esponenziali

equazioni esponenziali risolubili mediante applicazione delle proprietà delle potenze		
1	$6^x = 36$	$R: x = 2$
2	$49^x = \frac{1}{7}$	$R: x = -\frac{1}{2}$
3	$\left(\frac{1}{3}\right)^{x+1} = 9$	$R: x = -3$
4	$5^{x^2} = \frac{1}{25}$	$R: impossibile$
5	$8^{x^2-3x} = 1$	$R: x = 0 \cup x = 3$
6	$7^{x^2+4x+3} = \frac{1}{7}$	$R: x = -2$
7	$a^x = -2$	$R: impossibile$
8	$\left(\frac{2}{3}\right)^{-3x} = \frac{27}{8}$	$R: x = 1$
9	$3^{3x} = \frac{1}{27}$	$R: x = -1$
10	$\left(\frac{7}{3}\right)^{-2x} = \frac{9}{49}$	$R: x = 1$
11	$4^{x+8} = \frac{1}{4^{2x-5}}$	$R: x = -1$
12	$243^{\frac{2}{x}-1} = 27^{2(x+1)}$	$R: x = -\frac{5}{2} \cup x = \frac{2}{3}$
13	$7^{x^2-4x} - 1 = 0$	$R: x = 0 \cup x = 4$
equazioni esponenziali risolubili mediante una variabile ausiliaria		
14	$2^{2x} + 2^{x+1} - 1 = 0$	$R: x = \log_2(\sqrt{2} - 1)$
15	$10^{2x} + 3 \cdot 10^x + \frac{5}{4} = 0$	$R: impossibile$
16	$2e^{2x} + 5e^x = 3$	$R: x = \ln \frac{1}{2}$
17	$3^{2x} - 3^{x+1} = -2$	$R: x = 0 \cup x = \frac{\ln 2}{\ln 3}$
18	$20^{2x+1} + 2 = 9 \cdot 20^x$	$R: impossibile$
19	$\frac{3^{2x} - 3^{x+1}}{2} = -1$	$R: x = 0 \cup x = \frac{\ln 2}{\ln 3}$
20	$9^x - 3^{x+1} + 2 = 0$	$R: x = 0 \cup x = \frac{\ln 2}{\ln 3}$

Equazioni esponenziali

21	$5^{2\sqrt{x}} - 5^{\sqrt{x}+1} + 4 = 0$	$R: x = 0 \cup x = \log_5^2 4$
22	$3^{4\sqrt{x}} - 4 \cdot 3^{2\sqrt{x}} + 3 = 0$	$R: x = 0 \cup x = \frac{1}{4}$
23	$16^x - \frac{3}{2}4^x + \frac{1}{2} = 0$	$R: x = -\frac{1}{2} \cup x = 0$
24	$\left(\frac{1}{3}\right)^{x-1} + \left(\frac{1}{3}\right)^{1-x} = 2$	$R: x = 1$
25	$2^{3x-2} - 2^{3x-3} - 2^{3x-4} = 4$	$R: x = 2$
26	$3^{2-x} + 3^{x+1} = 12$	$R: x = 0 \cup x = 1$
27	$\frac{1}{7^{2x}} - \frac{1}{(7^x - 1)^2} = -\frac{2}{7^x - 7^{2x}}$	$R: x = -\frac{\ln 2}{\ln 49}$
28	$4^{x+8} = \frac{1}{4^{2x-5}}$	$R: x = -1$
equazioni esponenziali con basi diverse risolubili mediante logaritmi		
29	$3^x = 5^{3(x+1)}$	$R: x = \frac{2 \ln 5}{\ln 5 - \ln 3}$
30	$3^x = 5^{x-2}$	$R: x = \frac{2 \ln 5}{\ln 5 - \ln 3}$
31	$\frac{5^{2x}}{7^x} = 3$	$R: x = -\frac{\ln 3}{\ln 7 - \ln 25}$
32	$20 \cdot 7^x - 3 \cdot 2^x = 4 \cdot 7^x + 2^x$	$R: x = -2 \frac{\ln 2}{\ln 7 - \ln 2}$
33	$10^{x+3} - 2^{x+4} = 10^{x+2} - 2^x$	$R: x = -\frac{\ln 60}{\ln 5}$
34	$2^x \cdot 3^x = 10$	$R: x = \frac{1}{\log_{10} 6}$
35	$25^x \cdot 7^{x-1} = 2$	$R: x = \frac{\ln 14}{\ln 175}$
36	$\frac{3}{4} \cdot 5^x + 7 \cdot 3^x = \frac{2}{3} \cdot 5^x + 10 \cdot 3^x$	$R: x = \frac{2 \ln 6}{\ln 5 - \ln 3}$
37	$3^{2x-1} + 3^{2x-1} = 2 \cdot 5^{2x-1}$	$R: x = \frac{1}{2}$
38	$3 \cdot 7^x + 4 \cdot 3^x = 7^x + 10 \cdot 3^x$	$R: x = \frac{\ln 6 - \ln 2}{\ln 7 - \ln 3}$
39	$5^{x+1} + 2^{x+1} = 3 \cdot 5^x + 5 \cdot 2^x$	$R: x = \frac{\ln 3 - \ln 2}{\ln 5 - \ln 2}$
40	$\frac{2}{3^{x-1}} = 5^{3(x+1)}$	$R: x = \pm \frac{\sqrt{3}}{3} \sqrt{\frac{\ln 1125}{\ln 5}}$

Equazioni esponenziali

equazioni esponenziali di riepilogo		
41	$\sqrt[x]{25} = \left(\frac{1}{5}\right)^{x-4}$	$R: x = 2 \pm \sqrt{2}$
42	$2^{5x} \cdot 4^x = 1$	$R: x = 0$
43	$3(2^x + 1)^2 - 2(2^{2x} - 1) = 5(2^x - 1)^2$	$R: x = 2$
44	$4^x \sqrt{3^{2-x}} = 3^x \sqrt{2^{4x-1}}$	$R: x = \frac{2 \ln 108}{3 \ln 3 + 16 \ln 2}$
45	$3^{4-x} = \sqrt[5]{81^{3x-1} \cdot 27^x}$	$R: x = \frac{6}{5}$
46	$\frac{5 - 5^x}{5 + 5^x} - \frac{5 + 5^x}{5^x - 5} = -\frac{10}{3}$	$R: x = \frac{1}{\log 5}$
47	$a^{x-3} = \sqrt[x]{a^{10}}$	$R: x = -2 \cup x = 5$
48	$2^{3x+1} + 5^{2x+1} = 2^{3x+2} + 5^{2x}$	$R: x = \frac{\ln 2}{\ln 8 - 2 \ln 5}$
49	$\frac{5}{3} 3^x = 3 \frac{5^{2x-1}}{5^x}$	$R: x = 2$
50	$\frac{(2^x + 5)^2}{5} - 5 = 2^x(2^x - 4)$	$R: x = \log_2 \frac{15}{2}$
51	$5^{x^2-7x+12} = 1$	$R: x = x = 3 \cup x = 4$
52	$\frac{10^{\frac{2x+2}{x}}}{3^x} = 1$	$R: x = \frac{1 \pm \sqrt{1 + \log 9}}{\log 3}$
53	$(\sqrt{a})^{x-1} = (a^{-1})^{x^2-x}$	$R: x = -\frac{1}{2} \cup x = 1$
54	$3^{4x+2} - 37 \cdot 3^{2x} + 4 = 0$	$R: x = -1 \cup x = \frac{\ln 2}{\ln 3}$
55	$10^{2\sqrt{x}-1} \cdot 0.1^{\sqrt{x}} = 4$	$R: x = (1 + \log 4)^2$
56	$3^{1-2x} - \frac{13}{3^x} = 3^{x+1} - 13$	$R: x = 0 \cup x = \pm 1$
57	$\frac{5^{\sqrt{3x+10}}}{5^x} = 5^4$	$R: x = -3 \cup x = -2$
58	$25^{\sqrt{x+1}} = 5^{\sqrt{2x-5}}$	$R: impossibile$
59	$e^{\frac{x-1}{x^2}} = 1$	$R: x = 1$
60	$\left(\frac{3^x + 3}{3^x - 4}\right)^2 - 5\left(\frac{3^x + 3}{3^x - 4}\right) = 0$	$R: x \neq \frac{\ln 4}{\ln 3} \cup x = \frac{\ln 23 - \ln 4}{\ln 3}$

2

Raccolta esercizi Equazioni esponenziali

Franco Fusier - 2011

RIPASSO-APPROFONDIMENTO
Esercizi sulle equazioni esponenziali

- 1) $9^x - 12 \cdot 3^x + 27 = 0$ - sol: $x = 1$; $x = 2$
- 2) $4^x - 3 \cdot 2^x + 2 = 0$ - sol: $x = 0$; $x = 1$
- 3) $4^x - 14 \cdot 2^x - 32 = 0$ - sol: $x = 4$
- 4) $9^x - 28 \cdot 3^x + 27 = 0$ - sol: $x = 0$; $x = 3$
- 5) $25^x - 30 \cdot 5^x + 125 = 0$ - sol: $x = 2$; $x = 1$
- 6) $3 \cdot 9^x - 4 \cdot 3^x + 1 = 0$ - sol: $x = 0$; $x = -1$
- 7) $4^{x+1} - 33 \cdot 2^{x-1} + 2 = 0$ - sol: $x = 2$; $x = -3$
- 8) $9^{x+1} + 8 \cdot 3^x - 1 = 0$ - sol: $x = -2$
- 9) $16^x - 16 \cdot 4^x = 0$ - sol: $x = 2$
- 10) $4^x + 3 \cdot 2^x + 2 = 0$ - sol: *impossibile*
- 11) $32 \cdot 4^x + 127 \cdot 2^x - 4 = 0$ - sol: $x = -5$
- 12) $5 \cdot 3^x - \frac{9^x}{2} - \frac{9}{2} = 0$ - sol: $x = 0$; $x = 2$
- 13) $-2 \cdot 4^x + 16 \cdot 2^x - 32 = 0$ - sol: $x = 2$
- 14) $25^x - 26 \cdot 5^{x-1} + 1 = 0$ - sol: $x = 1$; $x = -1$
- 15) $-12 \cdot 36^x - 10 \cdot 6^x + 2 = 0$ - sol: $x = -1$
- 16) $98 \cdot 49^x - 4804 \cdot 7^x + 98 = 0$ - sol: $x = 2$; $x = -2$
- 17) $8^x - 3 \cdot 4^x + 3 \cdot 2^x - 1 = 0$ - sol: $x = 0$
- 18) $27^x - 9^{x+1} - 3^x + 9 = 0$ - sol: $x = 0$; $x = 2$
- 19) $16 \cdot 64^x + 47 \cdot 16^x - 67 \cdot 4^x + 4 = 0$ - sol: $x = 0$; $x = -2$

3

Raccolta esercizi ***Equazioni esponenziali***

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Risolvere le seguenti equazioni esponenziali

- 01) $\left(\frac{2}{3}\right)^x + 4 = 5\left(\frac{3}{2}\right)^x$ [0] 02) $3^x - 6 = 3^{3-x}$ [2]
- 03) $15\left(\frac{3}{5}\right)^{2x} - 34\left(\frac{3}{5}\right)^x + 15 = 0$ [1 ; - 1] 04) $4^x + 2^{x+1} - 8 = 0$ [1]
- 05) $3^{2x} - 82 \cdot 3^x + 81 = 0$ [0 , 4] 06) $5^{2x} + 125 = 6 \cdot 5^{x+1}$ [1 , 2]
- 07) $5^x + 1 = 6 \cdot 5^{-x}$ [$\log_5 2$] 08) $2\left(\frac{7}{2}\right)^x + 7\left(\frac{2}{7}\right)^x = 9$ [0,1]
- 09) $2^x + 2^{x-1} + 2^{x-2} + 2^{x-3} + 2^{x-4} = 62$ [5] 10) $36^x - 10 \cdot 2^x \cdot 3^{x+1} = 216$ [2]
- 11) $2^{x+3} + 2^{x+2} + 2^{x+1} + 2^x - 30 = 0$ [1] 12) $2^{x+1} + 2^x = 24$ [3]
- 13) $6^{x-2} + 6^x = 222$ [3] 14) $3^{x+2} = 3^{x-1} + 26$ [1]
- 15) $2^{x+2} + 2^{x-1} = 48 + 3 \cdot 2^x$ [5] 16) $3^{x+1} + 3^{x-1} = 2 \cdot 5^x$ [1]
- 17) $3 \cdot 2^{x-2} - 2^x + 7^{x-2} = 0$ [2] 18) $(3^{2x} - 3)^2 - 3^{2x}(2 - 3^{2x}) = 3$ [0 , $\frac{1}{2}$]
- 19) $\frac{2^{2x} - 2^{x+2}}{2^x - 4} = 8$ [3 , (2)] 20) $\frac{2^{x+3} - 60}{2^x - 4} = 1$ [3]
- 21) $5^{2x}(5^{2x} - 27) + (5^x + 5)^2 = 10 \cdot 5^x$ [0,1] 22) $27(3^{x-3} - 3) = 81 \cdot 3^{-x} - 1$ [4]
- 23) $\frac{2^{\sqrt{x+1}} \cdot 2^{\sqrt{2x-5}}}{8} = 1$ [3] 24) $\frac{3^{\sqrt{x^2+16}}}{3^{\sqrt{6x+7}}} = \frac{27}{3^x}$ [$-\frac{7}{6}$, 3]
- 25) $\frac{4^{1+\sqrt{x}} + 4^{1-\sqrt{x}} - 10}{2^x - 8} = 0$ [$\frac{1}{4}$] 26) $\frac{2}{9^x - 1} = \frac{2}{3^x - 1} - \frac{3}{4}$ [1]
- 27) $\frac{2^{2x}}{5 \cdot 2^x - 2 \cdot 3^x} = \frac{3^{2x}}{3^{x+1}}$ [0 , 1] 28) $\frac{3^{x+1}}{3^{2x}} = \frac{2 \cdot 5^x + 5 \cdot 3^x}{5^{2x}}$ [1]
- 29) $3^{x-1} = \frac{2^{2x}}{3^{x+1}}$ [0] 30) $3^{x-\frac{1}{4}} = \frac{9}{3^{x-\frac{7}{4}}}$ [2]
- 31) $\frac{4}{4^x + 2^x - 2} + \frac{2^x + 3}{2^x + 2} = \frac{2^x + 2}{2^{x+1} - 2}$ [$\frac{1}{2}$] 32) $\left(\frac{2}{3}\right)^{2x-1} = \frac{3 \cdot 2^{x+1} - 2 \cdot 3^x}{3^{x+1}}$ [1]
- 33) $2^{4x} - 2^{3x} - 2^{x+2} + 4 = 0$ [0 , $\frac{2}{3}$] 34) $3^{3x} - 3^{2x} - 3^{x+1} + 3 = 0$ [0 , $\frac{1}{2}$]

35) $10^{2x} - 4 \cdot 5^{2x} - 2^{2x} + 4 = 0$ $(0, 1)$ **36)** $3 \cdot 2^{2x} = 2(4^x + 1)(1 - 4^x)$ $(-\frac{1}{2})$
37) $(3^x - 2)^2 + 3^x \cdot (3^x - 1) = 7$ (1) **38)** $(2^x + 1) \cdot (2^{x+2} + 3) = 5$ (-2)
39) $2[3^x(3^x - 4) - 3^x + 2] = (1 - 3^x)(1 + 3^x)$ $(1; -1)$ **40)** $3^{x+2} + 3^{1-x} = 28$ $(1; -2)$
41) $3^{\sqrt{x}} + 3^{4-\sqrt{x}} = 18$ (4) **42)** $2^{x+2} + 3 = 2^{-x}$ (-2)
43) $2^{2x + \frac{3}{2}} + 2^{\frac{3}{2} - 2x} = 9$ $(-\frac{3}{4}, \frac{3}{4})$ **44)** $2^{1-x} - 2^{1+x} = 3$ (-1)
45) $2^{4-x} + 2^{x+1} = 33$ $(4; -1)$ **46)** $\frac{3^x}{2\sqrt{2}} = \frac{2^x}{3\sqrt{3}}$ $(-\frac{3}{2})$
47) $2^{2x} - 3 \cdot 2^x - 4 = 0$ (2) **48)** $4^x - 5 \cdot 2^{x+1} + 16 = 0$ $(1; 3)$
49) $3 \cdot 2^{4x} - 5 \cdot 4^x = 2$ $(\frac{1}{2})$ **50)** $3^x - 3^{x-2} = 2^{2x} - 2^{2x-1}$ (2)
51) $2^x + 3^{x-2} + 2^{x-3} = 3^{x-1} + 3^{x-4} - 2^{x-4}$ (4) **52)** $3^x + 3^{x+3} = 252$ (2)
53) $2^x + 2^{x+1} + 2^{x+2} + 2^{x+3} + 2^{x+4} = 31$ (0) **54)** $2^{x-1} + 2^{x-2} - 2^{x-3} = 10$ (4)
55) $3^x + 3^{x+1} + 3^{x+2} = 39$ (1) **56)** $\sqrt{2^{x^2+5}} = 2\sqrt{2 \cdot 2^x}$ $(1 \pm \sqrt{2})$
57) $x^{-1}\sqrt{2^{x-2}} \cdot x^{-2}\sqrt{2^{x-1}} = x^{-2}\sqrt{x-1}\sqrt{2^{2x-1}}$ (3) **58)** $\frac{\sqrt[3]{2^{x+2}}}{(\sqrt{2^x})^{1-x}} = 2^{x^2}$ $(-\frac{4}{3}, 1)$
59) $\sqrt[4]{3^{x^2}} \cdot \sqrt[3]{\frac{3^x}{9}} = \sqrt[4]{27^{x^2-4}} \cdot \sqrt[6]{3^{x+4}}$ $(2; -\frac{5}{3})$ **60)** $(\sqrt{3^{x-3}})^x = \frac{3}{3^x}$ $(-1; 2)$
61) $6^x - 9 \cdot 2^x = 2 \cdot 3^x - 18$ $(1; 2)$ **62)** $3^{x-\frac{7}{4}} = \frac{4 \cdot 3^{x-1} - 3}{3^{x-\frac{1}{4}}}$
63) $(2^x \cdot \sqrt{2} - 4)(2^{x+1} - 1)(2^x + 2) = 0$ $(-1; \frac{3}{2})$ **64)** $(\frac{2}{7})^x - (\frac{2}{7})^{x+1} = 5$ $(\log_{\frac{2}{7}} 7)$
65) $(\sqrt[3]{\sqrt[3]{2^x} \cdot \sqrt{2}})^x = \sqrt[4]{\frac{2^{x^2-2x}}{4}} \cdot \sqrt[3]{\frac{2^{x+2}}{\sqrt{2}}}$ $(3, -\frac{3}{5})$
66) $\frac{1}{5^x + 1} + \frac{5^x}{5^x + 2} = \frac{17}{5^x(5^x + 3) + 2}$ $(\log_5 3)$ **67)** $10^x(10^{2x} + 5) = 6(10^{2x} - 2)$
 $(\log 3, \log 4)$
 $\frac{1}{\sqrt[3x]{9^{1+x}}} = \left(3^{\frac{1+x}{3x}}\right) \cdot \sqrt[3]{5^{1-x}}$ $\frac{1}{\sqrt[3x]{4^{1+x}}} = \left(2^{\frac{1+x}{3x}}\right) \cdot \sqrt[3]{3^{1-x}}$

4

Raccolta esercizi ***Disequazioni esponenziali***

Franco Fusier - 6/2011

Risolvere le seguenti disequazioni esponenziali

01) $3^x + \frac{1}{3 \cdot 3^x} > \frac{28}{9}$ [$x < -2 \vee x > 1$]

02) $7 \cdot 49^x - 50 \cdot 7^x + 7 > 0$ [$x < -1 \vee x > 1$]

03) $2^{2x} - 5 \cdot 2^x + 4 < 0$ [$0 < x < 2$]

04) $(3^{x+1})^{x-1} - 3^{2x^2-1} \cdot 3^{2-x^2} + 216 > 0$ [$-2 < x < 2$]

05) $\sqrt[3]{5^{x+1}} > 25$ [$0 < x < 1$] 06) $8^x - 3 \cdot 4^x > 2^{x+2} - 12$ [$x < 1 \vee x > \frac{\log 3}{\log 2}$]

07) $2^{\frac{11}{2x+3}} > 32^{\frac{1}{2-x}}$ [$-\frac{3}{2} < x < \frac{1}{3} \vee x > 2$]

08) $4^x + 2^{x+1} - 3 > 0$ [$x > 0$]

09) $25^x - 5^x > 0$ [$x > 0$]

10) $2^{x+1} + \frac{8}{2^x} > 17$ [$x < -1 \vee x > 3$]

11) $\sqrt[3]{2^9} - 2^x < 0$ [$-3 < x < 0 \vee x > 3$]

12) $3^{2x} - 108 \cdot 3^x + 2187 < 0$ [$3 < x < 4$]

13) $3^{4x-1} - 81 > 0$ [$x > \frac{5}{4}$] 13) $2 \cdot 2^x > 5 - 4^{x-1}$ [$x > 1$]

14) $2^x + 2^{x+1} - 2^{x-1} > 20$ [$x > 3$]

15) $\frac{2 \cdot e^x - 1}{e^x - 3} > 0$ [$x < -\ln 2 \vee x > \ln 3$]

16) $\frac{3^{x+1}}{27^{2x}} < \frac{1}{3^{x^2+5}}$ [$2 < x < 3$]

17a) $3^{x+2} < 2^{4x+8}$ 17) $\sqrt{2^x} \geq 8\sqrt[3]{4^{x-1}}$ [$x \leq -14$]

18) $2^{\frac{2x+4}{x}} < \left(\frac{1}{4}\right)^{-2}$ [$x < 0 \vee x > 2$]

19) $\frac{3^{x+1} - 3^{x-1}}{2 \cdot 3^x + 3^{2x} + 1} > \frac{1}{2}$ [$-1 < x < 1$]

20) $2^{x+1} < (4^x - 5 \cdot 2^x)^{\frac{1}{2}}$ [\emptyset]

21) $3^{x+2} + 3^x < 30$ [$x < 1$] 22) $3^{-x} > 3^{-1}$ [$x < 1$]

$$23) \frac{5^x}{5^x - 1} + \frac{3}{5^x + 1} < -\frac{2}{1 - 5^{2x}} \quad [\forall x \in \mathbb{R}]$$

$$24) \sqrt[3]{3 \cdot x^{+1}} \sqrt[3]{9} < 3 \quad [x < -1 \vee 1 - \sqrt{2} < x < 0 \vee x > 1 + \sqrt{2}]$$

$$25) \sqrt[3]{5 \cdot 2^x} \cdot x^{-1} \sqrt[3]{5} < 2 \quad [x < 0 \vee \frac{1}{2} < x < 1]$$

$$26) 3 \cdot \left(\frac{3}{4}\right)^{4x} - 7 \cdot \left(\frac{4}{3}\right)^{-2x} + 4 < 0 \quad [-\frac{1}{2} < x < 0]$$

$$27) \frac{55 \cdot 2^x}{5^{x+1}} - \frac{10}{5^{2x}} > 2^{2x} [x < 0 \vee x > 1]$$

$$28) \frac{2^{x+1}}{2^x - 1} - \frac{4}{2^x + 1} > \frac{8}{2^{2x} - 1} \quad [x < 0 \vee x > 1]$$

$$29) 3^{x+3} + 3^x - \frac{135}{3^{x-1}} > \frac{247}{3^{x-4}} \quad [x > 3]$$

$$30) \frac{4^x}{4^{x+1} - 4} + \frac{1}{16^x - 4^x} < \frac{1}{4^x - 1} \quad [x < 0]$$

$$31^*) \sqrt{2(5^x + 24)} - \sqrt{5^x - 7} \geq \sqrt{5^x + 7} \quad [\frac{\log 7}{\log 5} \leq x \leq 2]$$

$$32) 2^{4x} - 2^{3x+1} - 2^{2x} - 2^x + 1 - 2 > 0$$

$$33) 2^{2+x} - 2^{2-x} > 15 \quad 34) 5^{2x+1} > 5^x + 4 \quad 35) \sqrt{2^x} \geq 8 \cdot \sqrt[3]{4^{x-1}}$$

$$36) \frac{2^{x+1} - 7}{x - 1} < \frac{10}{3 - 2x} \quad 37) (4x^2 + 2x + 2)^{x^2 - x} > 1$$

$$38) \frac{1}{3^x + 3} < \frac{1}{3^{x+1} - 1} \quad 39) \sqrt{9^x - 3^{x+2}} > 3^x - 9$$

$$40) 4^x - 2^{2(x-1)} + 8^{\frac{2}{3}(x-2)} > 52$$

$$41) \frac{16^x - 5 \cdot 4^x + 4}{x - 3} < 0 \quad [x < 0 \vee 1 < x < 3]$$

$$42) \frac{6}{2^x - 1} + \frac{3}{2^x + 1} > \frac{2}{2^x - 1} + 5 \quad [0 < x < 1]$$

$$43) \frac{2^{2x} - 2^x - 2}{3^x - 3} > 0 \quad 44) \frac{5^x - 5}{3^{2x} - 3^x - 2} < 0$$